The Long – Short Strategy Based on Cointegration Concept

Elena Stângă, Flavia Anghel, Alina Avrigeanu

Abstract
This paper presents a strategy of asset allocation based on the concept of cointegration. The method used can be applied on non-stationary data and has the advantage of using the whole set of information given by the financial variables. The cointegration approach is used for the construction of portfolios that can accurately follow an index. The results obtained are used in order to construct a long-short strategy. The steps followed are: First, two new indices are constructed to mimic the evolution of the original index but one has a higher yield at the end of the period (“plus benchmark”) and the other has a lower yield (“minus benchmark”). The concept of cointegration is then used to build portfolios that follow as well as possible the new indices. The “plus portfolios” and “minus portfolios” constructed this way are combined in a long-short strategy that has a low correlation with the market returns.

Keywords: portfolio, cointegration, index tracking, stocks, long-short strategy

JEL Classification: E58, G11, G12

Introduction
Between the financial markets there are often strong links that can be operated by managers of portfolios for obtaining higher yields. To estimate the links between these markets, the concept of correlation is used very often. This measure however is relevant for short-term connections between assets and it offers no stable results over time. Thus, portfolios’ structures obtained on the basis of asset correlation must be changed each time the correlation is changing, thus generating additional costs of rebalancing the portfolios. To avoid these costs it is necessary to use a measure of long-term relations, such as the concept of cointegration. This concept has the advantage that allows the application of least square regression and maximum likelihood for non-stationary data series.

The cointegration concept has proved to be useful for the common trends’ investigation of time series and can be used successfully to model the dynamics of a system, both long and short term.

* Elena Stângă is Assistant Professor of Financial Markets at the Romanian American University in Bucharest. E-mail: elena.stanga@gmail.com.
Flavia Anghel is Associate Professor of Economic Analysis at the Romanian American University in Bucharest. E-mail: flavia.anghel@gmail.com.
Alina Avrigeanu is Associate Professor of Management at the Romanian American University in Bucharest. E-mail: alina.avrigeanu@gmail.com.
For the long-short strategy, two new indices are constructed based on the original index but one has a higher yield at the end of the period (“plus benchmark”) and the other has a lower yield (“minus benchmark”) at the same moment. The concept of cointegration is then used to build portfolios that follow as well as possible the new indices. The “plus portfolios” and “minus portfolios” constructed this way are combined in a long-short strategy that has a low correlation with the market returns.

**Methodology of the long-short strategy**

The long-short strategy means that an index tracking strategy is extend for further explorations of the tracking potential of the cointegrated portfolios.

First, two new indices are constructed to mimic the evolution of the original index but one has a higher yield at the end of the period (“plus benchmark”) and the other has a lower yield (“minus benchmark”). The concept of cointegration is then used to build portfolios that follow as good as possible the new indices. The “plus portfolios” and “minus portfolios” constructed this way are combined in a long-short strategy that has a low correlation with the market returns.

The two indices, “plus” and “minus” benchmarks are constructed by adding to and subtracting from the benchmark returns an annual excess return that will be uniformly distributed to daily returns.

The constructed portfolios that track the “plus” and the “minus” benchmarks must pass the cointegration test. Like in the index tracking strategy the estimation it is realized using OLS. Now there are regressed the new “plus” and “minus” indices prices on the portfolio stocks prices over the same calibration period as that of the first strategy. In this case, there are two cointegration regressions, written as:

\[
\text{index}_{\text{plus}}_t = a_1 + \sum_{i=1}^{n} a_{i+1} * p_{i,t} + \varepsilon_t
\]

\[
\text{index}_{\text{minus}}_t = b_1 + \sum_{i=1}^{n} b_{i+1} * p_{i,t} + \varepsilon_t
\]

Based on this strategy a long position will be taken on the “plus” portfolio and a short position on the “minus” portfolio.

Another assumption that must be done is that the stock weights have no restriction of being positive in the tracking portfolios. This leads the opportunity to take some short positions for both “plus” and “minus” portfolios.

After the new indices and the new tracking portfolios are created the Engle – Granger methodology is applied in order to test the cointegration existence. Like in the first strategy, the portfolios created have 3, 4 or 5 stocks; the calibration period is maximum three years; the rebalancing was the same, meaning that the cointegration coefficients were rebalanced every 10 trading days and the number of stocks was kept constant.
For the long-short strategy there were followed the same steps as for the index tracking strategy, meaning: the tracking error was computed for each of the “plus” and “minus” tracking portfolios and the following statistics were estimated: the annual and daily volatility, the correlation of the tracking errors with the “plus” and “minus” benchmarks, skewness and kurtosis.

In addition to the tracking error, the returns of the long-short strategy were also computed as the difference between the returns of the “plus” portfolio and the returns of the “minus” portfolio. The statistics that were used for the tracking error were also used for the strategy returns in order to analyze its properties.

Data

The strategy described above is applied to a series of six Romanian stocks that are traded on the Bucharest Stock Exchange and are part of the BET index. These six stocks were selected because they have a long time series with very few missing values; they have high market liquidity and a high liquidity.

The daily stock closing prices were downloaded from Bucharest Stock Exchange and the missing observations were replaced by the last closing price available (the previous value of the series or the next value if data at the beginning of the series is missing). The data series include the stock prices from 03.01.2002 until 21.12.2007, and it has a total of 1483 observations.

The index was constructed by giving each stock an equal weight in the index and the price series of the equities were normalized.

The reasons for constructing a different index and not using directly the BET index are: First, the BET index uses the market capitalization of the stocks as a variable in determining the weight of each equity in the index. Because of this variable, in order to replicate the index you have to follow the history of the index adjustments due to corporate events, fact that would complicate the study. Second, the composition of the BET index changed over time, and for some of the current stocks we do not have enough past data in order to estimate the cointegration relationships.

Back test and results

This strategy uses the initial market index and builds two new indices that mimic the benchmark, but at the end of the period one has a higher yield and the other one has a lower yield compared to the initial index. After the new indices are created, the methodology mentioned above is used in order to construct portfolios that track the “plus” and “minus” indices.

Those two indices, “plus” and “minus” benchmarks are constructed by adding to and subtracting from the original index an annual excess return equal to 20%. This excess return is uniformly distributed to the daily returns of the “plus” and “minus” indices.
The “plus” and the “minus” benchmarks are presented in the figure no 1:

![Graph showing the 'plus' and 'minus' indices]

**Figure no 1 – The ‘plus’ and the ‘minus’ indices**

After the new two indices had been constructed there were created the new portfolios using OLS. In this strategy the new “plus” and “minus” indices are regressed on the prices of stocks over a calibration period of three, two years or one year.

Nine portfolios were constructed in order to track the ‘plus’ benchmark and other nine portfolios were constructed in order to track the ‘minus’ benchmark.

This strategy implies that a long position is taken on the “plus” portfolio and a short position is taken on the “minus” portfolio.

After the new indices and the new tracking portfolios are created the Engle – Granger methodology it is applied in order to test the cointegration relationship. The portfolios created have three, four or five stocks; the same calibration period of maximum three years was used; the portfolios were rebalanced every ten trading days and the number of stocks was kept constant.

The rebalancing allows the structure of the portfolios to change when the stocks prices are changing so they can track the “plus” and the “minus” benchmarks.

For this strategy, four portfolios track the ‘plus’ index well and also four portfolios have good results in tracking the ‘minus’ index.

The following figures show how the portfolios change their structure due to the changes in the stock prices.
The long – short strategy based on cointegration concept

Figure no 2 - portfolio no 1 with three stocks and one year of calibration that tracks the ‘plus’ index

Figure no 3 - portfolio no 1 with three stocks and one year of calibration that tracks the ‘minus’ index

By comparing the two graphics we notice that the parameters of this portfolio are good both for tracking the ‘plus’ index and the ‘minus’ index.

The next two portfolios (portfolio no 2, which has three stocks and two years of calibration, and portfolio no 3, that has also three stocks, but three years of calibration) have the same performance as those presented above. They follow almost to the same precision the ‘plus’ index and the ‘minus’ index.

A difference in tracking the ‘plus’ and the ‘minus’ benchmarks is noticeable for the last portfolio (portfolio no 7). This portfolio is constructed based on five stocks
and has only one year of calibration.

Figure no 4 - portfolio no 7 with five stocks and one year of calibration that tracks the ‘plus’ index

Figure no 5 - portfolio no 7 with five stocks and one year of calibration that tracks the ‘minus’ index

In this case it is obvious that the tracking portfolio follows better the ‘minus’ index than it follows the ‘plus’ one. This difference appears because in the first case the portfolio over – performs the ‘plus’ index and it can not track the index as it did in the beginning. This is caused also by the fact that stocks prices have increased together in the last part of the analyzed period.

For the long-short strategy the same statistics were computed as for the index
tracking strategy: volatility (daily and annual), skewness and kurtosis; the statistics are calculated both for the portfolios that track the ‘plus’ benchmark and the portfolios that track the ‘minus’ benchmark.

The results of these statistics are displayed in the next tables and they were estimated, in total, for 18 portfolios:

<table>
<thead>
<tr>
<th></th>
<th>Port 1</th>
<th>Port 2</th>
<th>Port 3</th>
<th>Port 4</th>
<th>Port 5</th>
<th>Port 6</th>
<th>Port 7</th>
<th>Port 8</th>
<th>Port 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ystd</td>
<td>0.24</td>
<td>0.17</td>
<td>0.15</td>
<td>0.22</td>
<td>0.16</td>
<td>0.13</td>
<td>0.16</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>Std</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.58</td>
<td>1.20</td>
<td>0.94</td>
<td>1.18</td>
<td>1.72</td>
<td>1.39</td>
<td>1.24</td>
<td>1.11</td>
<td>1.16</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>19.2</td>
<td>27.3</td>
<td>30.6</td>
<td>16.1</td>
<td>32.0</td>
<td>43.7</td>
<td>31.9</td>
<td>34.6</td>
<td>76.8</td>
</tr>
<tr>
<td>correlation</td>
<td>-0.15</td>
<td>-0.10</td>
<td>-0.13</td>
<td>-0.06</td>
<td>-0.09</td>
<td>-0.12</td>
<td>-0.07</td>
<td>-0.04</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

Table no. 1 – the tracking errors’ statistics for the ‘plus’ benchmark

<table>
<thead>
<tr>
<th></th>
<th>Port 1</th>
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<tr>
<td>Ystd</td>
<td>0.24</td>
<td>0.19</td>
<td>0.16</td>
<td>0.23</td>
<td>0.23</td>
<td>0.18</td>
<td>0.14</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>Std</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.74</td>
<td>1.09</td>
<td>0.87</td>
<td>2.08</td>
<td>1.53</td>
<td>1.12</td>
<td>2.16</td>
<td>1.25</td>
<td>0.82</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>18.9</td>
<td>20.2</td>
<td>25.9</td>
<td>22.5</td>
<td>21.2</td>
<td>29.2</td>
<td>39.9</td>
<td>42.7</td>
<td>67.0</td>
</tr>
<tr>
<td>correlation</td>
<td>-0.15</td>
<td>-0.09</td>
<td>-0.12</td>
<td>-0.01</td>
<td>-0.07</td>
<td>-0.09</td>
<td>-0.02</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

Table no. 2 – the tracking errors’ statistics for the ‘minus’ benchmark

From those results it can be concluded that the third portfolio and the seventh one are the best choices because they have the lowest volatility (portfolio no 3: 15.23% for the ‘plus’ index and 15.84% for the ‘minus’ one and portfolio no 7: 16.12% for the ‘plus’ index and 15.18% for the ‘minus’ one) and also the tracking errors are not correlated with the market.

The tracking errors have also a positive skewness with values starting from 0.5824 (for ‘plus’ index) and 0.735 (for ‘minus’ index) and reach 1.722 – for ‘plus’ and 2.081 – for ‘minus’. This means that the distribution of portfolios appear to be a non-normal distribution.

The kurtosis has also positive and high values (kurtosis greater than 3), starting at 16.05 – for ‘plus’ and 18.91 – for ‘minus’ and reaching 76.78 for ‘plus’ and 67.01– for ‘minus’. This is also a distribution that is more outlier-prone than the normal distribution (leptokurtic distribution).

The final stage of this strategy is to compute the returns of the “plus” portfolio (for long positions) and for the “minus” portfolio (for the short positions). The returns of the portfolios are calculated with the original index as a reference, in order to be comparable. Next, the return of the strategy is computed as the difference
between the “plus” portfolio returns and the “minus” portfolio returns for each pair of portfolios with the same parameters. In total, the returns of nine long-short strategies are analyzed.

The results show that only a single long-short strategy has a positive return in the long-run, the other eight strategies are not efficient.

It is interesting that the effective strategy is that where the portfolios are composed of three stocks and have a single year of calibration.

The following figure shows the results of the efficient strategy:

Figure no 6 – Strategy no 1 – the efficient long–short strategy

As it can be observed in the figure, even this strategy does not have only good results, it has also negative returns for a period. In the same time we notice that the returns of the strategy also have a high volatility, but it is not the highest one if we compare it to the other strategies.

The result of the analysis is not surprising if we take into consideration that the Romanian stock market can be classified as an emerging market, and thus the long-run relationship between the stocks is not so strong.

The performance of the strategy can also be analyzed with the help of the computed statistics: the volatility, the correlation, the skewness and the kurtosis. These statistics are presented in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Port 1</th>
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<th>Port 7</th>
<th>Port 8</th>
<th>Port 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ystd</td>
<td>0.06</td>
<td>0.07</td>
<td>0.04</td>
<td>0.12</td>
<td>0.09</td>
<td>0.07</td>
<td>0.11</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>Std</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.44</td>
<td>-2.1</td>
<td>-2.14</td>
<td>-5.45</td>
<td>-2.79</td>
<td>-1.7</td>
<td>-8.8</td>
<td>-0.59</td>
<td>-0.65</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>22.2</td>
<td>24.3</td>
<td>21.9</td>
<td>91.8</td>
<td>34.0</td>
<td>19.1</td>
<td>186</td>
<td>10.6</td>
<td>8.79</td>
</tr>
<tr>
<td>correlation</td>
<td>0.01</td>
<td>-0.17</td>
<td>-0.10</td>
<td>-0.32</td>
<td>-0.17</td>
<td>-0.15</td>
<td>-0.32</td>
<td>-0.18</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

Table no. 3 – the statistics for the returns of the long – short strategy
These statistics confirm that this strategy performs well:
- the strategy has one of the lowest volatility (6% - annual);
- strategy returns are not correlated with the market (it has the lowest value of correlation, equal to 0.01);
- the kurtosis value (which is equal to 22.2) is moderate compared to the other strategies, but the distribution is more outlier-prone than the normal distribution (leptokurtic distribution)
- the skewness has a moderate negative value, which means that the data is spread out more to the left of the distribution.

Conclusions

As expected, the results were not so well for this strategy. The long-short strategy implies that the portfolios should follow artificial indices, thus the cointegration relationship is weaker compared to the relationship with the original index. This aspect was also proved by the fact that only one in nine long-short strategies had a positive result at the end of the period.

In the same time, the result of the analysis is not surprising if we take into consideration that the Romanian stock market can be classified as an emerging market, and thus the long-run relationship between the stocks is not so strong.

Further research can be done in order to identify a stock selection method that gives better results. Research can also be done for the identification of a methodology that can help identify ex-ante the portfolios that have a good potential. Finally, the analysis could also be improved by taking into account longer data series and more stocks in the portfolios.

References


Bucharest Stock Exchange website www.bvb.ro