

SOME TYPICAL SHAPES OF HYSTERETIC LOOPS USING THE BOUC-WEN MODEL

Ovidiu Solomon¹

ABSTRACT

Bouc-Wen model possesses an appealing mathematical simplicity. It has the ability to portray the hysteretic behaviour of materials and devices. Using the Bouc-Wen equation for different values of loop parameters and imposed cyclic loading, typical shapes of hysteresis loops are obtained. Typical shapes of hysteresis loops are obtained using the Bouc-Wen equation for different values of loop parameters and imposed cyclic loading.

Keywords: Bouc-Wen model, hysteretic loops, system behaviour

1. 1. INTRODUCTION

The Bouc-Wen model, generally used in structural and mechanical engineering, gives an analytical description of a smooth hysteretic behavior. It was introduced by Bouc [1] and extended by Wen [2] who demonstrated its versatility by producing a variety of hysteretic characteristics. The hysteretic behavior of materials, structural elements or vibration isolators is treated in a unified manner by a single nonlinear differential equation with no need to distinguish different phases of the applied loading pattern [3]- [6].

In practice, the Bouc-Wen model is mostly used in the following inverse problem: given a set of experimental input–output data, how to adjust the Bouc-Wen model parameters so that the output of the model matches the experimental data. Once an identification method has been applied to find the Bouc-Wen model parameters, the resulting model is considered as a good approximation of the true hysteresis when the error between the experimental data and the output of the model is small enough from practical point of view [7].

Using this simple differential model proposed by Bouc(1967) and generalized in (1976) by Wen, a variety of hysteretic loops are obtained. In this model, the hysteretic force z is obtained for an imposed cyclic displacement x and is described by a nonlinear differential equation [3] – [6]:

$$\dot{z} = -\gamma |\dot{x}| z |z|^{n-1} - \beta \dot{x} |z|^n + A \dot{x} \quad (1)$$

or equivalent

$$\dot{z} = [A - |z|^n (\beta + \gamma \operatorname{sgn}(\dot{x}z))] \dot{x} \quad (1')$$

¹ PhD, solomonovidiu@yahoo.com

where A , β , γ , n are loop parameters controlling the shape and magnitude of the hysteresis loop.

A smooth transition can't be obtained if n tends to infinity. In this case, the obtained loops is very similar with the one obtained by the elasto-plastic hysteretic model.

2. THE BOUC-WEN MODEL (N NATURAL NUMBER)

Typical shapes of hysteretic loops are obtained using Bouc-Wen model for different values of loop parameters and a imposed displacement $x(t)=x_0\sin(2\pi ft)$ where the frequency $f=1$ Hz and the amplitude $x_0=1$ mm [8].

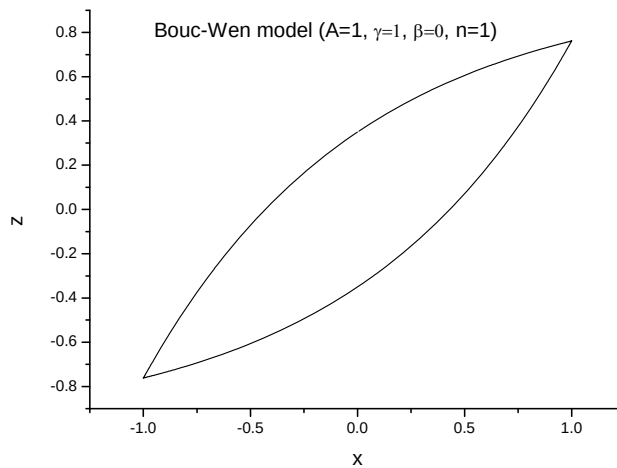


Fig. 1. Linear hysteretic behaviour

The linear hysteretic behaviour can be seen in figure 1. The hysteretic behaviour with degradation (Fig. 2) respectively with consolidation (Fig. 3) can be modeled using Bouc-Wen model.

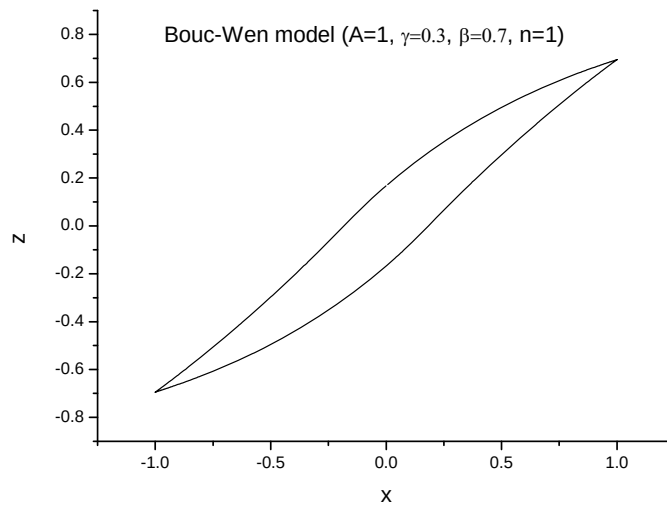


Fig. 2. Hysteretic behaviour with degradation (softening)

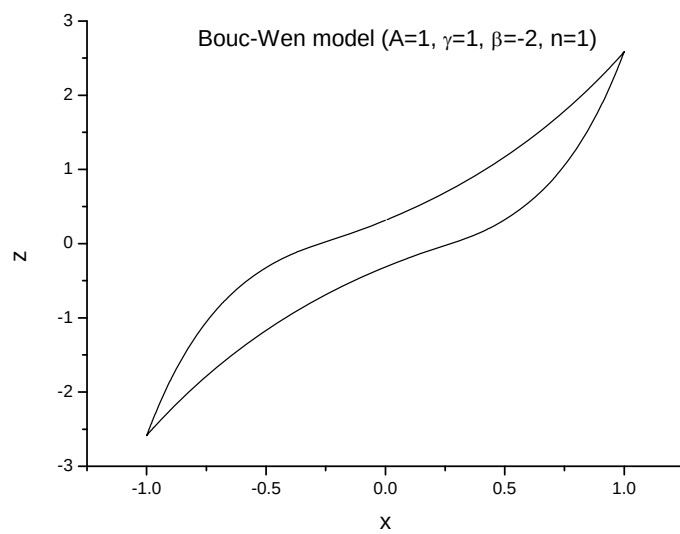


Fig. 3. Hysteretic behaviour with consolidation (hardening)

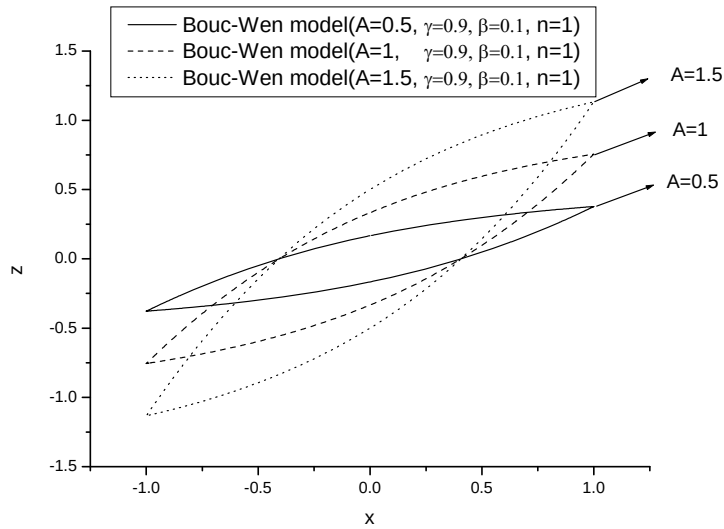


Fig. 4. Variation of hysteretic loop with parameter A

Parameter A controls the scale and the amplitude of the hysteretic curved controls and the slope variation of the stiffness characteristic (Fig. 4). The Bouc-Wen model must reproduce the property of energy dissipation for an adequately representation of physical behavior of the real systems. If the value of the parameter A is negative, then the mechanical energy of the system is not dissipated; therefore, this is not the physical behavior of a real system.

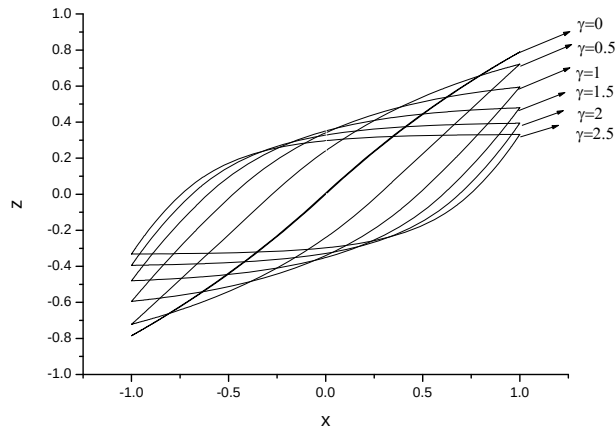


Fig. 5. Variation of hysteretic loop with parameter γ

If the parameters $\beta = 0.5$, $n=1$, $A=1$ are set, then the variation of parameter γ has the effect of turning the hysteresis loop in clockwise, increasing the area of the hysteresis loop and

curved of backbone. For $\gamma = 0$ the area of hysteretic loop is zero, then increases, giving a peak in the range $\gamma \in [1, 1.5]$ after that the area starts to decrease.

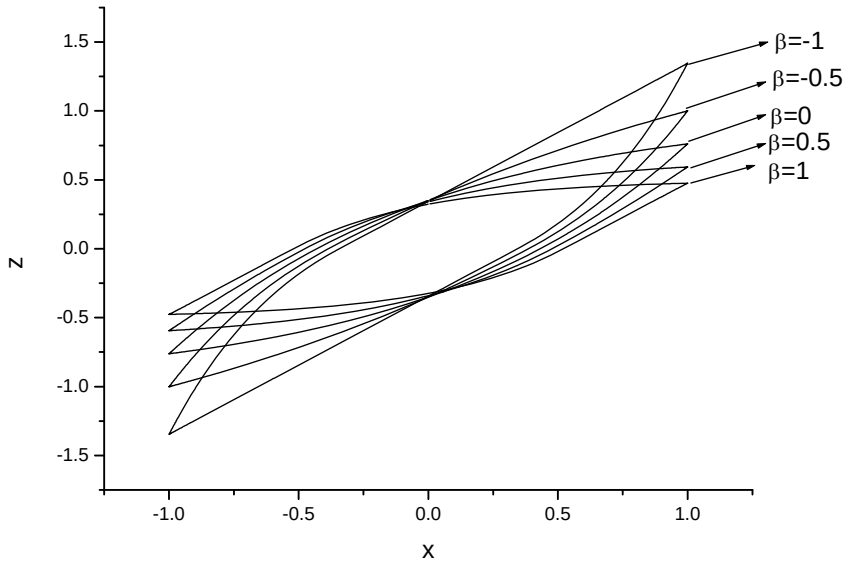


Fig. 6. Variation of hysteretic loop with parameter β

The parameter β controls the shape of the hysteretic curve:

$\beta = 0 \Rightarrow$ linear hysteretic behaviour

$\beta < 0 \Rightarrow$ hardening hysteretic behaviour

$\beta > 0 \Rightarrow$ softening hysteretic behaviour

The variation of parameter β for $A=1$, $\gamma=1$, $n=1$ has the effect of turning the backbone of hysteresis loop in clockwise simultaneously with its curving clockwise for negative values of the parameter β and clockwise for positive values of the same parameter.

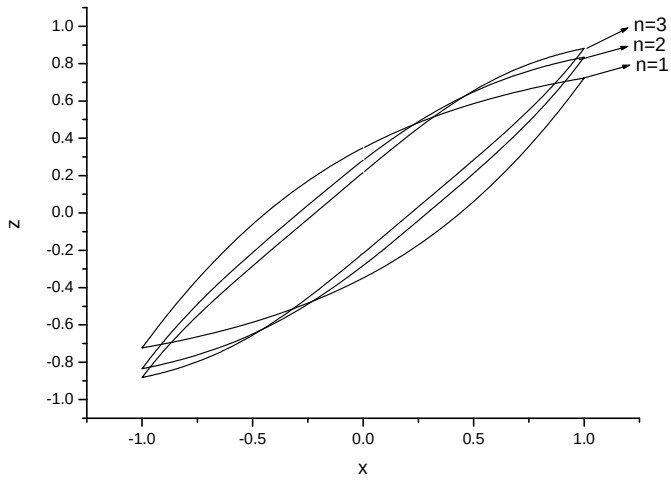


Fig. 8. Variation of hysteretic loop with parameter n (natural)

The parameter n controls the shape of the hysteretic curve. The effect is important between 1 and 2 values. For $n > 2$ the differences are less important.

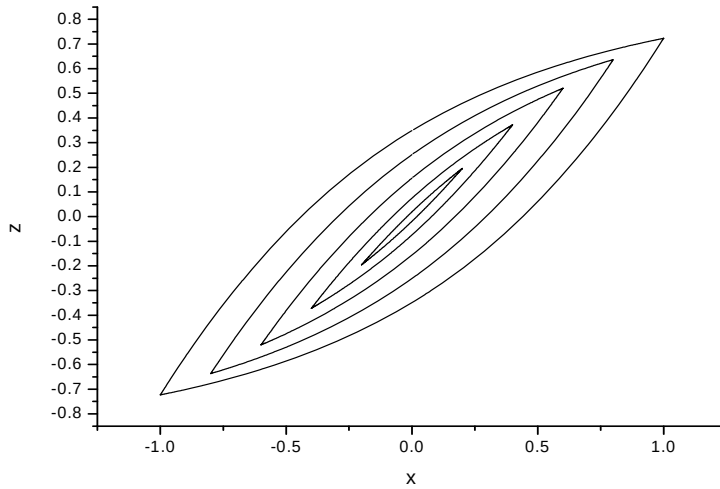


Fig. 8. Variation of hysteretic loop with amplitude

The variation of hysteretic loops in respect to amplitude x_0 is shown in fig. 8, where the parameters of hysteretic loops are $A=1, \gamma=1, \beta=0.1, n=1$ and x_0 takes values: 0.2, 0.4, 0.6, 0.8, 1.

3. THE BOUC-WEN MODEL (N POSITIVE NUMBER)

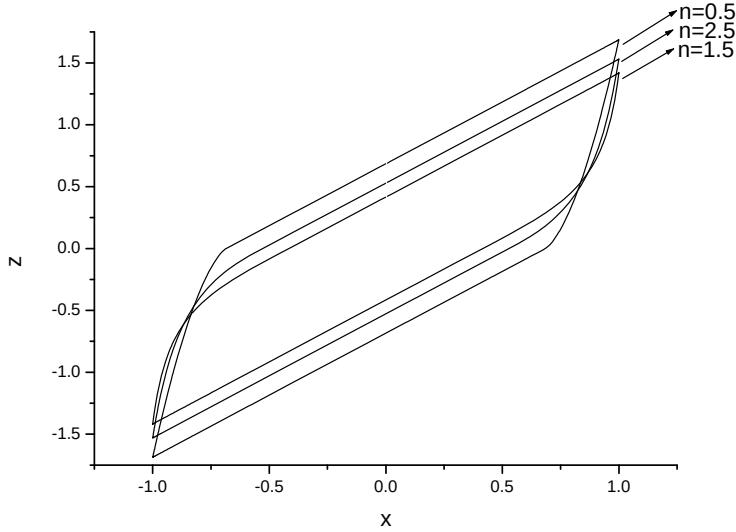


Fig. 10. Variation of hysteretic loop with parameter n (positive)

Taken $A=1, \gamma=3, \beta=-3$ and $n=0.5, n=1.5, n=2.5$ the hysteretic loops is shown in figure 10. As one can see from figure 10, the area of hysteretic loops decreases as the parameter n increases.

4. THE EXTENDED BOUC-WEN MODEL

Since the Bouc-Wen model does not describe classical hysteretic loops containing an inflection point in the cyclic loading, an extended Bouc-Wen model with five parameters was proposed [9]:

$$\dot{z} = \left[A - \beta |z|^p - \gamma |z|^q \operatorname{sgn}(z \dot{\xi}) \right] \dot{\xi} \quad (2)$$

where p și q are positive real numbers.

This inflection point appears in the experimental hysteresis loops. It corresponds to a hysteretic force changing from a softening characteristic to a hardening one during a sequence of cyclic loading [9].

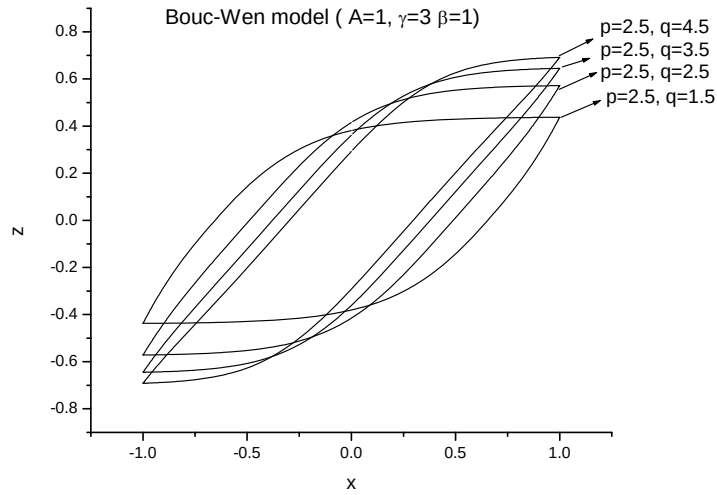


Fig. 11. Variation of hysteretic loop with parameter q

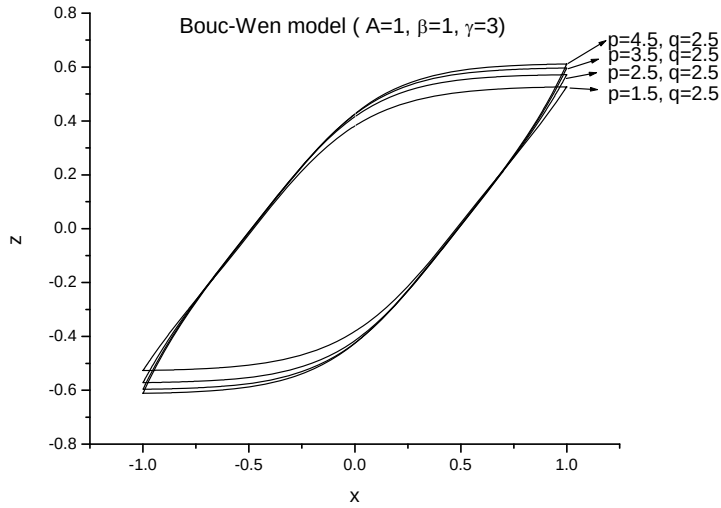


Fig. 12. Variation of hysteretic loop with parameter p

Figures 11 and 12 show the variation of hysteretic loop with parameter q respectively p and $A=1, \gamma=3, \beta=1$. In the first case, when q increases, the area of hysteretic loops decreases. In the second case, while p increases, the area of the hysteretic loops decreases.

5. CONCLUSIONS

The Bouc-Wen model has a remarkable versatility and capability to describe the force variation for any type of time history of the imposed device displacement or material strain.

Choosing appropriately the parameter values, one can control both the slope variation of the stiffness characteristic and the energy dissipated by cycle.

The Bouc-Wen model captures a range of shapes of hysteretic loops which resemble the properties of a wide class of real non-linear hysteretic systems.

6. REFERENCES

- [1] *Bouc, R.*, Forced vibration of mechanical systems with hysteresis, Proceedings of the Fourth Conference on Non-linear oscillation, Prague, Czechoslovakia, 1967.
- [2] *Wen, Y.K.*, Method for random vibration of hysteretic systems, Journal of the Engineering Mechanics Division, 102 (2), pp. 249–263, 1976.
- [3] *Mayergoyz I. D.*, Mathematical Models of Hysteresis and their Applications, Elsevier, New York, 2003.
- [4] *Macki, J. W., Nistri, P., and Zecca P.*, Mathematical Models for Hysteresis, *SIAM Review*, Society for Industrial and Applied Mathematics, V.35, No. 1, pp. 94-123, 1993.
- [5] *Ikhoulane F, Rodellar J, Hurtado JE*, Analytical Characterization of Hysteresis Loops Described by the Bouc-Wen Model, *Mech. of Adv. Mater. and Struct.*, 13, pp.463–472, 2006.
- [6] *Visintin A.*, *Differential Models of Hysteresis*, Springer, Berlin, 1994
- [7] *Giuclea M., Sireteanu T., Ghita Gh.*, Metode de inteligență computațională cu aplicații la sisteme dinamice, Ed. Bren, 2008.
- [8] *Mitu A. M., Giuclea M., Ghita G., Sireteanu T.*, Portraing the hysteretic behaviuor of materials and devices by Bouc-Wen models, International Symposium on Energy Dissipation. Acoustical, Vibratory and Seismic Processes, Bucharest, paper on CD-ROM Proceedings, 2005.
- [9] *Sireteanu T., Giuclea M., Mitu A.M.*, Identification of an extended Bouc-Wen model with application to seismic protection through hysteretic devices, *Computational Mechanics*, Springer Berlin, ISSN 0178-7675, 2010