

A STUDY ABOUT IMAGE NOISE REMOVAL

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ABSTRACT

This paper presents a comparison between different techniques of noise removal such as NL-means algorithm, local smoothing filters, bilateral filtering, wavelet thresholding and median filtering, based on current researches. The main goals of this article are to recover the original image after applying a noise measurement, smoothing the images and preserves the edges.

KEYWORDS: *bilateral filtering, NL-means algorithm, wavelet thresholding, median filtering*

1. INTRODUCTION

There are many methods proposed for removing the noise and recovering the real image, but all these methods have something in common: they involve an average (Gaussian smoothing model, anisotropic filtering, neighborhood filtering, and wavelet thresholding methods).

Paper [1] proposes a method of decomposition D which is expressed in equation (1), where „v” is the noisy image and „h” the filtering parameter. Authors propose a method which involves the average of all pixels in the image. A comparison is presented between the NL-means algorithm and the local smoothing filters. A solution for modeling the effect of noise for a digital image is using Gaussian noise. For the formula (1), the Gaussian value is $n(i)$.

$$v(i) = u(i) + n(i) \quad (1)$$

Paper [2] presents a method of bilateral filtering. The approach is smoothing the images and preserving the edges. It is non-iterative, simple and local. It works with gray levels or colors based on geometric closeness and photometric similarity. The method preserves edges for human perception. Bilateral filtering is different in comparison with other filters because it produces no phantom colors along edges for colored images, and reduces phantom colors in the original image.

In [7] the authors present median filtering algorithm for image noise reduction. Image filters are classified as linear and nonlinear. The median filter is a nonlinear digital filtering technique, often used to remove noise. The median filter is very effective in preserving edges in an image while reducing random noise. It is also useful in reducing an impulsive type of noise and particularly effective at removing ‘salt and pepper’ type of

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noise. To replace the value of each pixel with the median value of neighboring, the median filter has to move through the image pixel by pixel and calculate it sorting all the pixel values from the window into numerical order.

Under ideal conditions, the signals are without noise or, if they are affected by noise, this noise may be decreased to a negligible level while the signal increases. In reality, it is often needed to reduce/remove the noise corrupting a signal in order to recover the original signal. The noise removal can take place in time-frequency or time-scale. In the domain of signal processing, time-frequency analysis means that a signal is studied simultaneously in time and frequency domains (two dimensional signal) [8].

A basic form of time-frequency analysis is the short-term Fourier transform and a better method is using wavelets. Wavelet transform is among the most popular time frequency transformations. The transformation should not allow changes in shape but in time extension. The simplest type of wavelet is the Haar wavelet. Noise removal/reduction is the process that reconstructs the original signal from a signal with noise. Current methods are lacking in solving local structures. This is required when dealing with signals that contain structures of various sizes and amplitudes. The analysis and signal processing, speech and audio processing, as well as image processing, we often encounter nonlinear signals [8].

2. PROBLEM DEFINITION

Article [1] proposes a method of decomposition D which is (2), where „v” is the noisy image and „h” the filtering parameter. A formula is applied (3) which assumes that the value of “x” is a mean of the values which look like the ones in the neighborhood of x. It uses all possible self-predictions for the image. In the formula (3), it is a constant which is normalized, is a Gaussian kernel and „h” is a filtering parameter.

$$v = Dhv + n(Dh, v) \quad (2)$$

$$NL[u](x) = \frac{1}{C(x)} \int_{\Omega} e^{-\frac{(C_a * |u(z+.) - u(y+.)|^2)_{(0)}}{h^2}} u(y) dy \quad (3)$$

Article [2] presents a non-iterative scheme for the edge in a way of preserving smoothing. The scheme could be implemented by a single layer of neuron-like devices for one operation per image. First: a low pass domain filter is applied to the image and produces an output image defined in (4), where „c” is the geometric closeness between the center x and a nearby point.

$$h(x) = k_d^{-1}(x) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi, x) d\xi \quad (4)$$

In [7] the authors try to obtain performance in noise-reducing of the median filter:

$$\sigma_{med}^2 = \frac{1}{4nf^2(\underline{n})} \approx \frac{\sigma_t^2}{n + \frac{\pi}{2} - 1} \cdot \frac{\pi}{2} \quad (5)$$

$$\sigma_0^2 = \frac{1}{n} \sigma_t^2 \quad (6)$$

Paper [8] explains why wavelet transformation is a good option:

- The output of the wavelet transformation is maximal when the input signal is very similar to the mother wavelet because the transformation performs a correlation analysis
- If the energy of a signal is concentrated in a small number of wavelet dimensions, the coefficients will be considerably higher in comparison with a signal or noise that distributes the energy over many coefficients

The term of white noise means:

- The values of the noise aren't correlated in time
- Has equal power at all frequencies
- Affects every single frequency component over the whole length of the signal and for that is considered the most difficult to remove.

3. AVAILABLE METHODS

Paper [1] evaluates several classical local smoothing filters: Gaussian filtering [2], anisotropic filtering [3], the total variation minimization [4] and the neighborhood filtering [5]. Bilateral Filter is presented in [2]. In [7] and [8] are presented wavelet thresholding and median filtering.

Gaussian filtering

The approach is built around the Gaussian kernel $x \rightarrow G_h(x) = \frac{1}{(4\pi h^2)} e^{-\frac{|x|^2}{4h^2}}$, where „h” represents standard deviation. Equation (7) represents the theorem 1 and shows the convolution of the image method noise with and „h” is small enough.

$$u - G_h * u = -h^2 \Delta u + o(h^2) \quad (7)$$

The Laplacian cannot be small near edge or texture, the Gaussian noise is zero only in harmonic parts of the image.

The anisotropic filtering

The anisotropic filtering is convolving the image „u” at point „x” in the direction orthogonal to $Du(x)$. It is presented in (8), where $Du(x) \neq 0$ and G_h has variance h^2 .

$$AF_h u(x) = \int G_h(t) u \left(x + t \frac{Du(x)^\perp}{|Du(x)|} \right) dt \quad (8)$$

Equation (9) represents the image noise. With $curv(u)(x)$ we make note of the curvature.

$$u(x) - AF_h u(x) = -\frac{1}{2} h^2 |Du| curv(u)(x) + o(h^2) \quad (9)$$

The Total Variation minimization

The approach is based on a given noisy image $v(x)$, and aims to recover the original image $u(x)$. It uses formula (10) and theorem (11) where we calculated the Total Variation minimization. $TV(u)$ shows the total variation of „ u ”, γ is a given Lagrange multiplier.

$$\begin{aligned} TVF_{\lambda}(v) &= \arg \min TV(u) + \lambda \int |v(x) - u(x)|^2 dx - TVF_{\lambda}(u)(x) \\ &= -\frac{1}{2\lambda} \text{curv}(TVF_{\lambda}(u))(x) \quad (11) \end{aligned}$$

The neighborhood filtering

Neighborhood filters create a new pixel by taking an average of similar gray level value of neighboring pixels. Yaroslavsky (1985) makes an average of pixels with the similar gray level value which belongs to the spatial neighborhood $B_p(x)$ (12).

$$YNF_{h,p}u(x) = \frac{1}{C(x)} \int_{B_p(x)} u(y) e^{-\frac{|u(y)-u(x)|^2}{h^2}} dy \quad (12)$$

NL-means algorithm

In (13), v is the discrete noisy image, $NL[v](i)$ is the estimated value which is computed as an average of weights of all the other pixels. The family of weights is formed based on the similarity between the pixels "i" and "j". The similarity between two pixels i and j depends on the intensity of gray (v).

$$NL[v](i) = \sum_{j \in I} w(i,j) v(j) \quad (13)$$

The weights are defined in (14), where „ h ” is a filter parameter, and Z is a normalizing constant which is defined in (15).

$$w(i,j) = \frac{1}{Z(i)} e^{-\frac{\|v(N_i)-v(N_j)\|_2^2}{h^2}} \quad (14)$$

$$Z(i) = \sum_j e^{-\frac{\|v(N_i)-v(N_j)\|_2^2}{h^2}} \quad (15)$$

The NL-means compares the gray level in a single point and the entire configuration for the neighborhood.

NL-means consistency

$j \in \{1, \dots, n\}$, in (16) explains the conditional expectation theorem. The theorem shows that the algorithm is not trying to separate the noise, but rather correcting the noise in the image.

$$|NL_n(j) - r(j)| \rightarrow 0 \quad (16)$$

Bilateral Filter

Bilateral Filter [2] is a simple range filter applied to the window. The filtered image is the result of a local remapping and it is different at different points in the image. From (4), they obtain (17), so the filter is shift-invariant and „c” is only a difference function and is a constant.

$$k_d(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi, x) d\xi \quad (17)$$

Range filtering is defined in (18), where „s” measures the photometric similarity between the pixel at the neighborhood with center „x” and the center of the nearby point.

$$h(x) = k_r^{-1}(x) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) s(f(\xi), f(x)) d\xi \quad (18)$$

Then the normalization constant is replaced by the constant in (19).

$$k_r(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(f(\xi), f(x)) d\xi \quad (19)$$

They name as bilateral filtering the combination between domain and range filtering. Combined filtering is shown in (20), and the normalization is shown in (21)

$$h(x) = k^{-1}(x) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) c(\xi, x) s(f(\xi), f(x)) d\xi \quad (20)$$

$$k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi, x) s(f(\xi), f(x)) d\xi \quad (21)$$

Median Filtering

The “window” is the pattern of neighbors which slides pixel by pixel over the entire image. The output of median filtering is $g(x, y) = \text{med}\{f(x - i, y - j), i, j \in W\}$, where $f(x, y)$, $g(x, y)$ are the original image and the output image respectively. “W” is the 2-D mask. The mask size can be 3x3, 5x5, n x n and the shape may be square, circular, cross, linear etc.

Image Denoising using Wavelet Thresholding

When wavelet transform is linear wavelet denoising works fine for additive noise:

$$W(a, b)[f + n; \Psi] = W(a, b)[f; \Psi] + W(a, b)[n; \Psi];$$

The 3 steps of general method are:

- Step 1: Decompose signal using DWT;
- Step 2: Perform thresholding:
 - Finite length signal with additive noise:
 - $y[n] = x[n] + \sigma r[n]$, $0 \leq n \leq N-1$, where $x[n]$ - signal, σ - standard deviation, $r[n]$ - noise $\sim N(0,1)$, $N - 1 = 2^{J+1} - 1$ - dyadic sampling.

- Step 3: Threshold DWT coefficients for signal reconstruction: $\hat{x} = W^{-1} \hat{X}$

Thresholding Methods

- Hard Thresholding

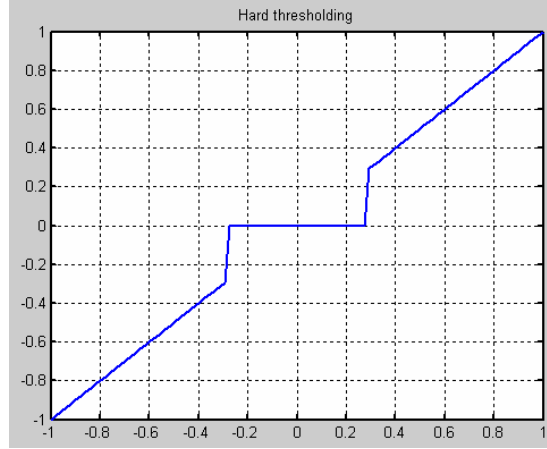


Figure 1. Image taken from [8]

$$y_{hard}(t) = \{x(t), \text{where } |x(t)| > \delta; 0, \text{where } |x(t)| < \delta\}$$

- Soft Thresholding

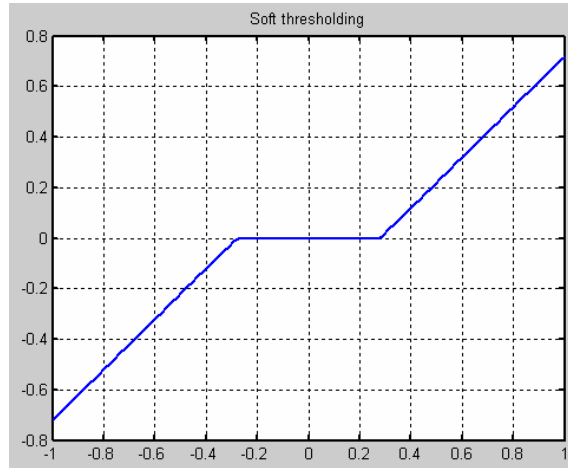


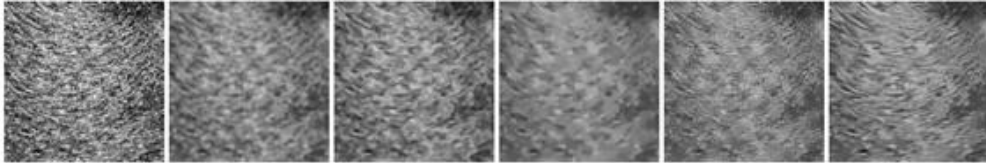
Figure 2. Image taken from [8]

$$y_{soft}(t) = \{sgn(x(t)) * (|x(t) - \delta|), \text{where } |x(t)| > \delta; 0, \text{where } |x(t)| < \delta\}$$

4. RESULTS AND DISCUSSIONS

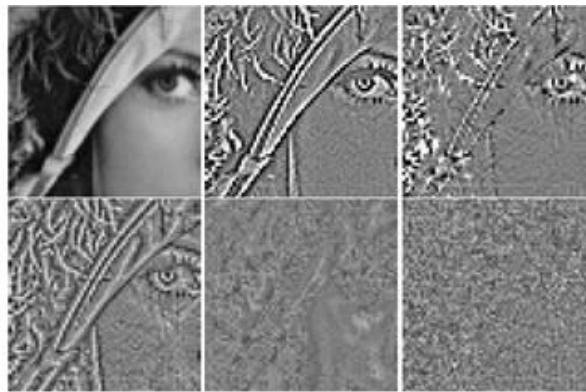
This section, compares the local smoothing filters, NL approach, bilateral filter, median filtering and wavelet thresholding.

Figure 3 displays a comparison of the results of the approaches presented. Fig. 3 presents a denoising experience for a natural texture and Fig. 4 presents an experience on a natural image.



Denoising experience on a natural texture. From left to right: noisy image (standard deviation 35), Gauss filtering, anisotropic filtering, Total variation, Neighborhood filtering and NL-means algorithm.

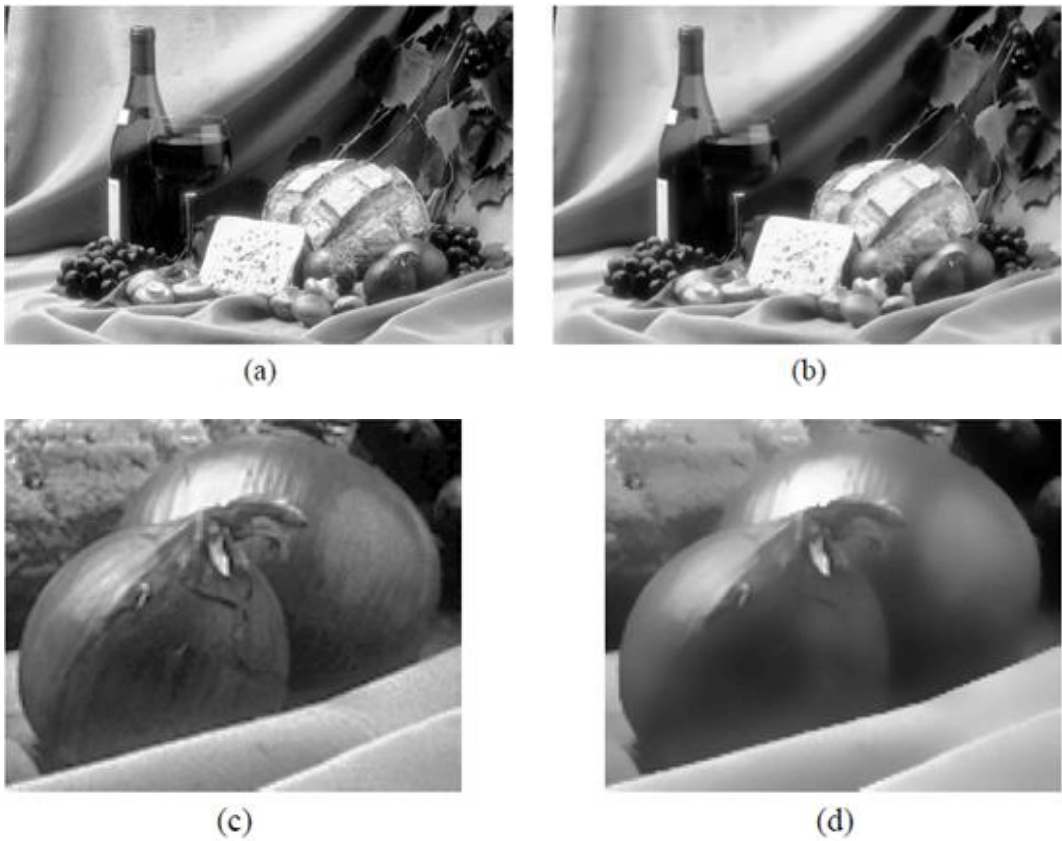
Figure 3. Image taken from [1]



Method noise experience on a natural image. Displaying of the image difference $u - D_h(u)$. From left to right and from top to bottom: original image, Gauss filtering, anisotropic filtering, Total variation minimization, Neighborhood filtering and NL-means algorithm. The visual experiments corroborate the formulas of section 2.

Figure 4. Image taken from [2]

Applying Bilateral Filter we can obtain the following pictures (Fig. 5).



A picture before (a) and after (b) bilateral filtering. (c,d) are details from (a,b).

Figure 5. Image taken from [2]

Results for median filtering techniques are the following:

- 1-D simple signal:

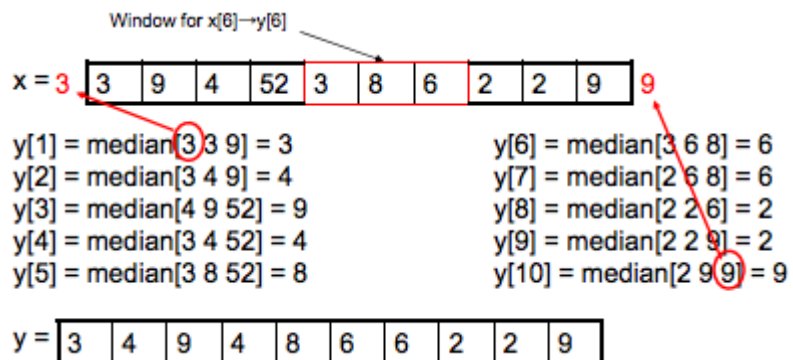


Figure 6. Image taken from [7]



Figure 7. Image taken from [7]

In Fig.7 the image on the left contains a significant amount of “salt and pepper” noise. After processing with a median filter we obtain the same image but without noise on the right.

- Simple two dimensional signal (2-D) - example using 3 x 3 sampling window:
 - If we keep border values unchanged:

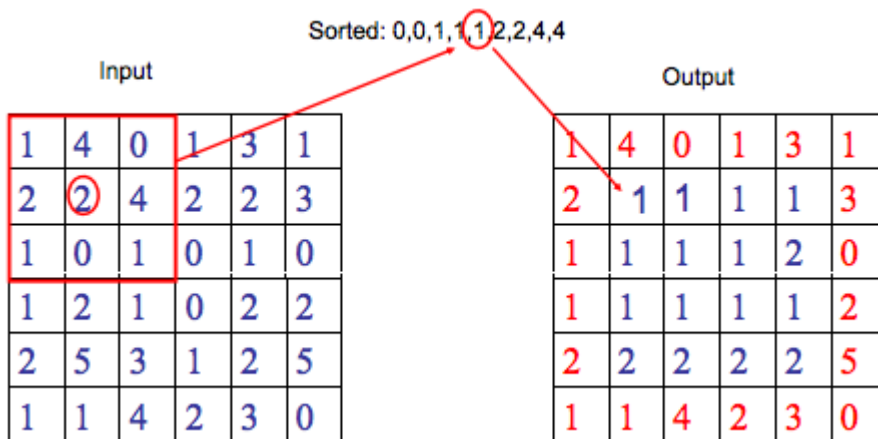


Figure 8. Image taken from [7]

- If we extend border values outside with values at boundary:

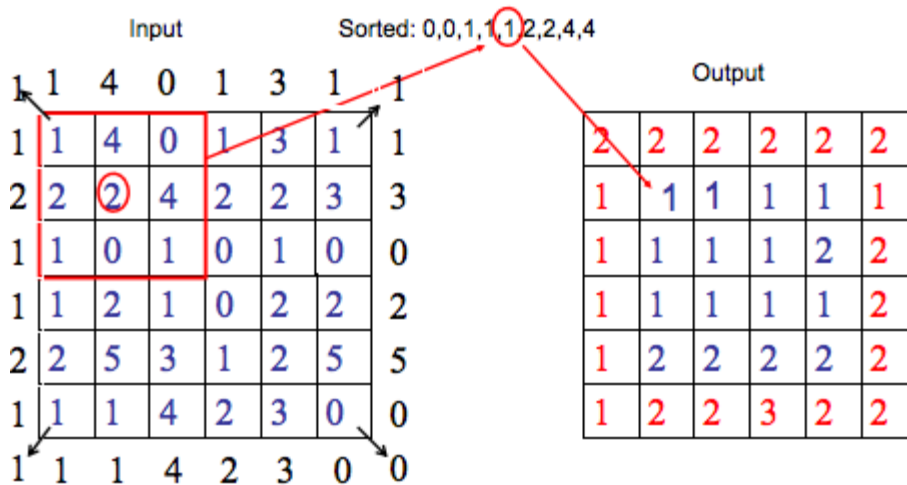


Figure 9. Image taken from [7]

- Extending border values outside with 0s:

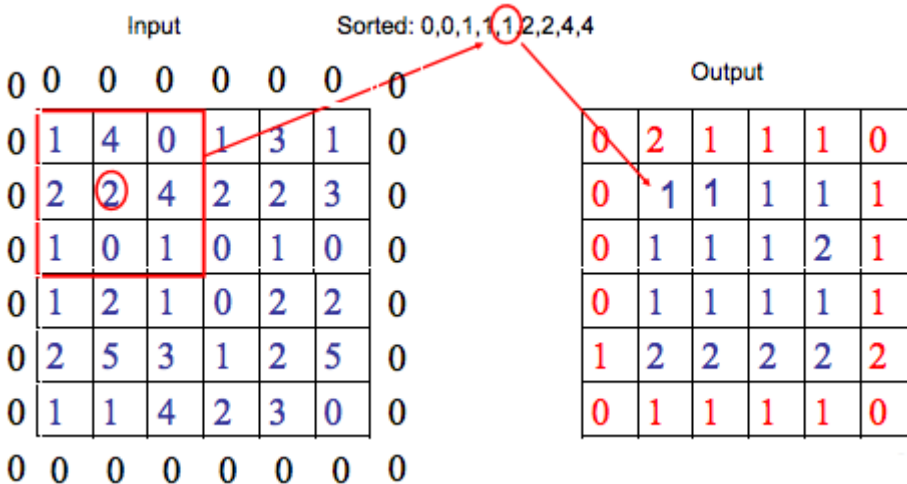


Figure 10. Image taken from [7]

In [8] are presented why, in some cases, it is good to apply the soft threshold to a few detail levels, and the hard one to the rest.



Figure 11. Image taken from [8]

As we can see in Fig. 11, the hard thresholding keeps the edges but doesn't remove much of the noise. On the other hand, soft thresholding removes almost all of the noise, but doesn't provide the same edge preservation.

5. CONCLUSIONS

This paper presents the current techniques used for noise removal such as NL-means algorithm, local smoothing filters, bilateral filtering, wavelet thresholding and median filtering. We presented how these techniques are used, the advantages for each one and some experiments that other researchers have made.

The advances in noise removal techniques may prove extremely useful in other noise-sensitive image analysis and processing areas like: binarization based on locality statistics [9], line or other primitive detection using Hough transform and data post-processing [10], OCR automatic text improvement [11] and layout analysis from image segmentation [12].

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