Time Delays and The Underwriting Cycle

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ABSTRACT: We shall consider the concept of time delays and the extent to which this is a common feature in many general insurance systems. We shall then present an example of a model of an insurance system with delays that helps to explain the phenomenon of underwriting cycles.

Keywords: insurance, time delays, underwriting cycle, forecasting, rating formula.

Delays in non-life insurance

The insurer's liability to pay a claim crystallizes at the time of the insured event. Many favors can lead to delays between the occurrence of the event and the actual payment of the claim Thus, Ackman et al. (1985)[1] identify five sets of factors:
1. The event covered by the insurance policy may not occur at a single instant - for example, workmen's compensation claims arising from industrial disease may relate to exposure over a long time period and may not be recognized as claimable events until many years have elapsed since the inception of the policy.
2. There may be delays before a claimable event is reported to the insurer.
3. The legal liability of the insurer may not always be dear-cut, and there may be considerable delays before the situation is clarified (possibly involving the courts).
4. It may not be possible to determine the magnitude of the claim until some time after the occurrence of the insured event - for example, in motor damage claims there may be a delay until the vehicle can be examined and the damage assessed, and more extreme examples may arise in personal injury cases which involve the courts.
5. There may be processing delays within the insurer's administration departments, in recording the necessary statistical information on the claim, managing and updating of the claims file, and payment of the claim.

Time delays are introduced also by the regulatory process (see, for example, Lemaire 1985)[10]. Insurance premium rates are regulated in many countries. Insurance companies may be required to have their rates ed by regulatory authorities prior to use (as in the US) or to approved by follow a uniform, national tariff (as in Switzerland). Regulation almost always creates additional delays between the experience period and the effective date of application of the revised rates. In addition, premium rates may be revised less frequently than under a competitive system (Cummins and Outreville, 1987)[7].

The process of collection and analysis of the data, the projection into the future and the effect of system delays have been well illustrated by Coutts (1984)[6], who demonstrates that, although motor insurance premiums may only be expected to be in force for one year and the length of each contact is only one year, the premium estimation process can involve projections of up to eight years. The presence of time delays in notification and settlement of claims and in data collection leads to this effect. Such an extension of the time frame under consideration then necessitates a number of important, subjective decisions, for example (Coutts, 1984)[6].

The underwriting cycle

A number of autors (for example, Cummins and Outreville (1987)[7] and Venezian (1985) [14]) have noted the existence of a cycle with a period of about six years in the profits of non-life insurance companies (or property-casualty companies, as they are named in North America).
The usual explanation for the existence of the cycle is that it is caused by increased profits leading to increased capably, leading to aggressive marketing and a decline in underwriting standards. This then leads to reduced profits, a decline in capacity, stricter underwriting and increased profits, and so the cycle is repeated Several other competing hypotheses have been proposed in the literature to explain the cycle: for example, Cummins and Outreville (1987)[7], Rantala (1988)[11] and Daykin et at. (1994)[9] provide more detailed reviews. For example, Venezian (1985)[14] puts forward an explanation in which he assumes that premiums as determined independently of the market by an insular, but he proposes a relationship between insurer behavior and the existence of cycles which is based on projections. Specifically Venezian points out that premium rating, at least as practiced in the US, relies on extrapolating past claim costs in order to predict future claim costs. These extrapolations tend to involve an estimation period of approximately three years, and an extrapolation period of about two years (Cummins and Outreville, 1987)[7].

Model based on time delays

We now turn to the specification of a model for explaining the existence of underwriting cycles. We exclude any consideration of expenses, taxes, investment income, interaction with the capital market or the methodology for fixing the premium rate. The main hypotheses is that the dynamics of the cycle come from the fact that profits feed back into surplus (or reserves) with a time delay.
A second-order autoregressive equation will generate a six-year cycle with particular values of the parameters, as in Venezian's (1985) analysis (see above). We shall describe a simple model that leads to such an equation and thus provides one possible explanation for the phenomenon of the underwriting cycle (see Berger, 1988[5], Daykin et al , 1994[9]). To reach this goal, it is necessary to assume two one-year delays in the structure of the business. In the presentation here, we follow the argument of Berger (1988)[5].
Thus, we assume that:
1. the insurer sets its underwriting policy for the forthcoming year on the basis of the end-of-year surplus (or reserves), so that the more financially secure is the insurer, the more walling it will be to underwrite the more marginal risks,
2. the profit and loss results follow from the underwriting policy with a one-year time delay;
3. the profit and loss results are passed directly into the surplus (or reserves) so that there are no distributions to policyholders or shareholders,
4. the effects of investment income, expenses and other cash flows can be honor.

We let P_t, Q_t, S_t and p_t respectively present market premium (or price), quantity, surplus (reserves) and economic profits for year *t*.
Given 1, we assume that the market supply or quantity is a function of the surplus in the immediately preceding period (since, as Berger (1988)[5] has shown, insurers will be more willing at any premium to underwrite marginal risks on the surplus is increased).
The resultant market premium and quantity will also depend on the surplus in immediately preceding period (since the position of the supply function will determine the intersection of the supply and demand functions): $(P_t, Q_t) = f(S_{t-1})$ for dome *f*.
We also assume (from 2.) that the profit in year *t* is a function of the premium (price) and quantity in year

t - 1, i.e.

$$p_t = g(P_{t-1}, Q_{t-1}) = gf(S_{t-2}) = h(S_{t-2})$$

for some functions g and h.
We thus have, from 3.,

$$S_t = p_t + S_{t-1} \quad (1)$$

by definition, and

$$p_t = h(S_{t-2}) = h(S_{t-3} + p_{t-2}) \quad (2)$$

If *h* is invertible, we have a second-order difference equation, possibly non-linear, for *p*, namely:

$$p_t = h(h^{-1}(p_{t-1}) + p_{t-2}) \quad (3)$$

When h is linear, $h(S) = aS + b$ and $h^{-1}(p) = (p-b)/a$, so we have:

$$p_t = p_{t-1} + ap_{t-2} \quad (4)$$

which is a homogeneous difference equation with general solution $p_t = k_1 c_1^t + k_2 c_2^t$ where c_1 and c_2 are solutions of $c^2 = c + a$, and k_1 and k_2 depend on the initial conditions p_0 and p_{-1}. Thus,

$$p_0 = k_1 + k_2 \text{ and } p_{-1} = \frac{k_1}{c_1} + \frac{k_2}{c_2}.$$

If $1 + 4a < 0$ (i.e. $a < -\frac{1}{4}$), c_1 and c_2 are complex conjugates, $c_1 = re^{i\theta}$, $c_2 = re^{-i\theta}$, where $r^2 = -a$ and $2r\cos\theta = 1$ so that $\cos\theta = 1/2\sqrt{-a}$.
Then

$$\begin{aligned} p_t &= k_1 c_1^t + k_2 c_2^t \\ &= r^t\left[(k_1 + k_2)\cos t\theta + i(k_1 - k_2)\sin t\theta\right] \\ &= r^t\left[p_0 \cos t\theta + \left(\frac{2ap_{-1} + p_0}{2r\sin\theta}\right)\sin t\theta\right] \end{aligned}$$

on eliminating k_1 and k_2.
So

$$\begin{aligned} p_t &= r^t\left[p_0 \cos t\theta + \left(\frac{2ap_{-1} + p_0}{\sqrt{-4a-1}}\right)\sin t\theta\right] \\ &= \alpha r^t \sin(t\theta + \beta) \end{aligned}$$

for some constants α and β that depend on a and the initial conditions.
The sine function has the property that $\sin(t_0\theta + \beta) = \sin(t_0\theta + 2\pi + \beta)$ for some t_0, which is equivalent to a cycle. To identify the period of the cycle T, we note that we would require

$$(t_0 + T)\theta + \beta = t_0\theta + 2\pi + \beta$$

i.e.

$$T = \frac{2\pi}{\theta}.$$

Thus, p_t follows a cycle with period:

$$T = \frac{2\pi}{\theta} = \frac{2\pi}{\cos^{-1}(\frac{1}{2\sqrt{-a}})}$$

If $a = -1$, then $T = 6$ years. Also, if $a = -1$ we note that, consistent with a six year cycle, $p_t + p_{t-3} = 0$.

We note also that the magnitude of r determine whether the cycles have an increasing $(r > 1)$, decreasing ($r < 1$) or constant ($r = 1$) amplitude.

Model based on forecasting and rating formula

In this section, we consider how the use of certain premium rating formulae can lead to damped sinusoidal variations in premiums, loss ratios and solvency ratios even if the claims process does not originally contain such elements. The result again would be the phenomenon of the underwriting cycle. The discussion here is based on the model of Balzer and Benjamin (1980)[3].

An intrinsic feature of insurance systems is the delay before claims are notified and settled. In some classes of business, 25% of the incurred claims may be unreported and/or unpaid after two years, while the situation could be more extreme with long-tailed classes of business like liability insurance.

We let C_t be the claims incurred for year t, and $1-e$ be the proportion of premiums absorbed by expenses, so that:

$$p_t = eP_t - C_t. \quad (5)$$

For the fixing of P_t, we shall analyze the effects of a 'wait and see' strategy. We shall relate P_t to B_t, the base premium for year t, and include (an element of profit-sharing so that premiums are reduced in year t, if recent business has been profitable to the insurance company. In practice, most insurers would experience difficulties in having final figures from year $t-1$ available for use in year t. Also, they may be a sufficient number of unpaid claims to render those figures undesirable for the purposes of purposes of profit-sharing feedback. The 'wait and see' or time-delayed strategy would be equivalent to saying that, for L time periods after the premium is paid, the accumulated surplus is unreliable and so for year t the value from year $t-L$ should be used for the profit sharing 'feedback' formulae, i.e.,

$$P_t = B_t - dS_{t-L} \quad 0 < d < 1 \quad (6)$$

So, from equations 5 and 6 we have

$$p_t = kB_t - C_t - deS_{t-L}$$

and, from equation 1, then

$$S_t - S_{t-L} + deS_{t-L} = A_t \, (7)$$

where $A_t = eB_t - C_t$.

Equation 7 is a difference equation for S_t which can be solved to determine P_t via equation 6.

We note that if the solution for S_t converges as $t \to \infty$ to a specific value S and A_t similarly converges to A, then, in the limit

$$S = \frac{A}{de}. \, (8)$$

If we require S > 0 then we would simply require $A > 0$ (or $eB > C$). If the sequence converges in the long run, it will converge to a positive value if, in the long run, the amount of base premium left after deducting expenses is greater than the claim amount. Of course, this does not guarantee convergence. We note that $A = 0$ would lead to a steady state solution of S = 0, if convergence occurs.

We follow Balzer and Benjamin (1980)[3] and consider some specific choices for L.

If $L = 1$, equation 7 becomes

$$S_t - mS_{t-1} = A \text{ where } m = 1 - de \quad (9)$$

With $S_0 = 0$, the solution is

$$S_t = \sum_{j=0}^{t-1} A_{t-j} m^j = \sum_{j=0}^{t-1} (eB_{t-j} - C_{t-j}) m^j \quad (10)$$

If $L = 2$, equation 7 becomes

$$S_t - S_{t-1} + deS_{t-2} = A_t \quad (11)$$

A trial solution of the form $S_t = x^t$ yields the quadratic $x^2 - x + q^2 = 0$ where $q^2 = de$, which has roots $x_{1,} x_2 = 1 \pm i\sqrt{4q^2 - 1}/2$ assuming that $4q^2 > 1$. For convenience, we rewrite x_1 and x_2 as $x_1 = qe^{i\varphi}$ and $x_2 = qe^{-i\varphi}$ and $\tan\varphi = \sqrt{4q^2 - 1}$. Then with $S_0 = 0$, the solution to 11 becomes, after some algebraic simplification,

$$S_t = \sum_{j=0}^{t-1} A_{t-j} q^j \frac{\sin(j+1)\varphi}{\sin\varphi} \quad (12)$$

Typical values of e and d might be 0.8 and 0.5 so that m = 0.6, q = 0.6325 and φ = 0.659.

Some insight can be gained by observing the reaction of accumulated surplus to a single pulse of incurred claims, C_1. Consequently we put B_t = 0 and C_2 = C_3 = ... = 0, leaving C_1 = X, non-zero and positive. Under these conditions, for $L = 1$,

$$S_t = -m^{t-1}.X = -(0.6)^{t-1} X$$

which involves a simple decay factor of 0.6 per annum.

For L = 2,

$$S_t = -q^{t-1}\left(\frac{\sin t\varphi}{\sin\varphi}\right) X = \frac{-(0.6325)^{t-1}(\sin 0.659t)X}{0.612}$$

which is an oscillatory result with a period $T = 2\pi/\varphi = 9.5$ years and a decay factor of 0.6325 per annum.

For the case L = 1, the dynamic response of S_t to the isolated group of unpredicted claims is satisfactory although the effects of this disturbance still take approximately seven periods to be eliminated (for the case m = 0.6).

When the delay L is increased to two periods, the responses oscillatory and overshoots. No recovery of the loss is attempted for two periods. Then it is over-collected in the next four periods, resulting in the insurer having to repay some of it in the following periods. This is not a situation with which insured or insurer would be happy. The overall settling time is extended by about one or two periods relative to the L = 1 case (Balzer and Benjamin,1980)[3].

It is a general principle of control engineering that the introduction of time delays into a feedback loop leads to instability. When L = 5, numerical experiments with equation 7 show that the system becomes completely unstable watts ever-increasing oscillations in S_t. With S_t diverging as $t \to \infty$, it is clear that one or other of the parties to the contract would withdraw from the arrangement rather than suffer these dramatic oscillations.

The value of L does not necessarily imply a delay of L years. The use of quarterly feedback and a delay time of two years would correspond to L = 8, which again leads to high instability.

It is noteworthy that these particular results are quite general, and are independent of the type of insurance and the choice of base premiums. Further, stability and instability are properties of the system itself and are not related to the nature of the particular disturbance input we have used as an illustration.

These results have arisen from a positive 'spike' of unexpected claims. The insured may be more interested in the effect of lower than expected claims. This can be| similarly analyzed by considering the effect of a negative X as input.
Dagg (1995)[8] has exploded the properties of the model further by numerically analyzing the results for some more complicated cases, as follows.

Example 1

Consider *d=0.5* and *e=0.8* as before, and the effect of a stream of higher than expected claims i.e. $B_t = 1$ and $C_t = 1$ for each 1. Since $eB_t < C_t$, we would expect that, if convergence occurs, it leads to a negative long-run value. The results as shown in Figure1. We note that the amplitude of oscillation increases with the length of the lag, *L*, and that for $L = 1,...4$ the oscillations appear to be reducing over time and tending to a limit of about -0.5, as determined by equation 8. It is apparent that for L = 5 the curve is becoming increasingly unstable.

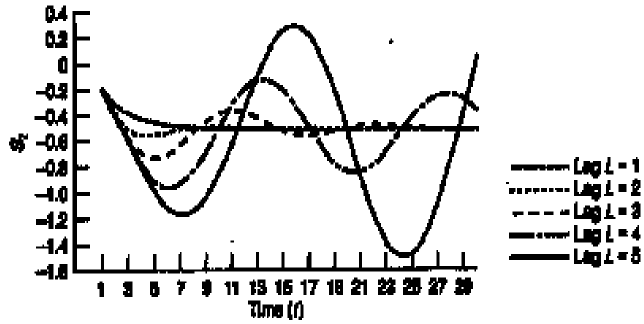

Fig. 1: Plot of accumulated surplus against time

Example 2

We consider $B_t = 1$ and $C_t = 1 + \sin(\pi t/3)$ so that the claims vary sinusoidally with a period of six years and upper and lower limits of 2 and 0 respectively. The patterns in Figure.2 for S_t are not as consistent as for Example 1. There is evidence that the amplitude increases with the delay L, but there is no clear evidence of convergence. We note that the increased complexity of the input has led to a more complex output.

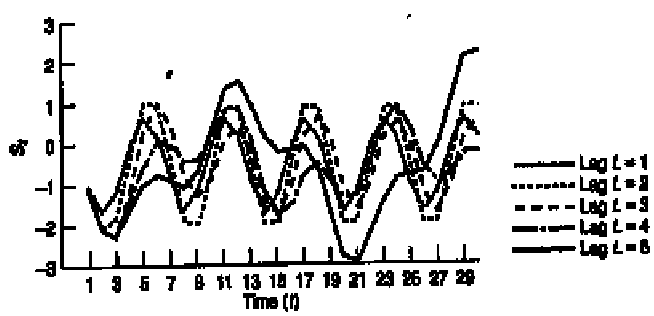

Fig 2. Plot of stimulated surplus against time.

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