Robust Monetary Policy

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ABSTRACT: While there is uncertainty about the data that enter into economic models and about the parameters that govern economic models, the fact that economists often approach macroeconomic data armed with different models of the economy suggests that uncertainty, or ambiguity, about the model could also be potentially important. A policy can be made "robust" to model uncertainty by designing it to perform well on average across all of the available fully specified models rather than to reign supreme in any particular model. In this paper we compare the implications of robust monetary policy versus non robust monetary policy for a model based on a new Keynesian model with two equations that represent the dynamics of inflation and the dynamics of the output gap. Using Matlab, we are able to approximate the solution to the linear–quadratic problem associated with the estimated model, thus obtaining the optimal monetary policy decision.

1. INTRODUCTION

According to Alan Greenspan (2003), "Uncertainty is not just an important feature of the monetary policy landscape; it is the defining characteristic of that landscape". In fact, the recognition that all monetary policymakers must bow to the presence of uncertainty appears to underlie Greenspan's (2003) view that central banks are driven to a "risk management" approach to policy, whereby policymakers "need to reach a judgement about the probabilities, costs, and the benefits of the various possible outcomes under alternative choices for policy".

Uncertainty comes in many forms. One obvious form is simply ignorance about the shocks that will disturb the economy in the future (oil prices, for example). Other forms of uncertainty, perhaps more insidious can also have resounding implications on how policy should be conducted, three of which are data uncertainty, parameter uncertainty, and model uncertainty.

2. THE MODEL

When solving robust control problems there are generally two distinct equilibria that are of interest. The first is the "worst-case" equilibrium, which is the equilibrium that pertains when the policymaker and private agents design policy and form expectations based on the worst-case misspecification and the worst-case misspecification is realized. The second is the "approximating" equilibrium, which is the equilibrium that pertains when the policymaker and private agents design policy and form expectations based on the worst-case misspecification, but the reference model transpires to be specified correctly.

According to the state – space formulation, the economic environment is one in which the behavior of an $n \times 1$ vector of endogenous variables, z_t, consisting of n_1 predetermined variables, z_{1t}, and $n_2 (n_2 = n - n_1)$ non predetermined variables, z_{2t}, are governed by the reference model

$$z_{1t+1} = A_{11} z_{1t} + A_{12} z_{2t} + B_1 u_t + C_1 \varepsilon_{1t+1}, \qquad (1)$$

$$E_t z_{2t+1} = A_{21} z_{1t} + A_{22} z_{2t} + B_2 u_t, \qquad (2)$$

where u_t is a $p \times 1$ vector of control variables, $\varepsilon_{1t} \approx iid[0, I_s]$ is an $s \times 1$ vector, $s \le n_1$, of white – noise innovations, and E_t is the mathematical expectations operator conditional upon information available up to and including period t. The reference model is the model that private agents and the policy maker believe most accurately describes the data generating process. The matrices $A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2$ contain structural parameteres and are conformable with z_{1t}, z_{2t} and u_t as necessary. The matrix C_1 is determined to insure that ε_{1t} has the identity matrix as its variance – covariance matrix.

The policymaker's problem is to choose a sequence for its control variables, $\{u_t\}_0^\infty$, to minimize the objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[z_t' R z_t + 2 z_t' U u_t + u_t' Q u_t \right], \tag{3}$$

where $\beta \in (0,1)$ is the discount factor. The weighting matrices, $R, U,$ and Q reflect the policymaker's preferences; R and Q are assumed to be positive semidefinite and positive definite, respectively.

Acknowledging that their reference model may be misspecified, private agents and the policymaker surround their reference model with a class of models of the form

$$z_{1t+1} = A_{11} z_{1t} + A_{12} z_{2t} + B_1 u_t + C_1 (v_{t+1} + \varepsilon_{1t+1}), \tag{4}$$

$$E_t z_{2t+1} = A_{21} z_{1t} + A_{22} z_{2t} + B_2 u_t, \tag{5}$$

where v_{t+1} is a vector of specification errors, to arrive at a "distorted" model. The specification errors are intertemporally constrained to satisfy

$$E_0 \sum_{t=0}^{\infty} \beta^t v_{t+1}' v_{t+1} \le \eta, \tag{6}$$

where $\eta \in [0, \overline{\eta}]$ represents the "budget" for misspecification.

3. ROBUST POLICYMAKING WITH COMMITMENT USING STATE – SPACE METHODS

In the commitment solution, both the policymaker and the evil agent are assumed to commit to a policy strategy and not succumb to incentives to renege on that strategy. Employing the definitions

$$\tilde{u}_t \equiv \begin{bmatrix} u_t \\ v_{t+1} \end{bmatrix}, \tilde{B} \equiv \begin{bmatrix} B & C_1 \end{bmatrix}, \tag{7}$$

$$\tilde{U} \equiv \begin{bmatrix} U & 0 \end{bmatrix}, \tilde{Q} \equiv \begin{bmatrix} Q & 0 \\ 0 & -\theta I \end{bmatrix} \tag{8}$$

the optimization problem can be written as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[z_t' R z_t + 2 z_t' \tilde{U} \tilde{u}_t + \tilde{u}_t' \tilde{Q} \tilde{u}_t \right], \tag{9}$$

subject to

$$z_{t+1} = A z_t + \tilde{B} u_t + \tilde{C} \varepsilon_{t+1}, \tag{10}$$

which, because the first – order conditions for a maximum are the same as those for a minimum, has a form that can be solved using the methods developed by Backus and Drifill ([1]). Those methods involve formulating the problem as linear – quadratic, the value function has the form $V(z_t) = z_t' V z_t + d$ and the dynamic program can be written as

$$z_t' V z_t + d = \min_{u_t} \max_{v_{t+1}} [z_t' R z_t + 2 z_t' \tilde{U} \tilde{u}_t' + \tilde{u}_t' \tilde{Q} \tilde{u}_t + \beta E_t (z_{t+1}' V z_{t+1} + d)]. \tag{11}$$

It is well known that the solution to this optimization problem takes the form

$$\begin{bmatrix} u_t \\ v_{t+1} \end{bmatrix} = -FT^{-1} \begin{bmatrix} z_{1t} \\ p_{2t} \end{bmatrix}, \tag{12}$$

$$z_{2t} = [V_{22}^{-1} V_{21} V_{22}^{-1}] \begin{bmatrix} z_{1t} \\ p_{2t} \end{bmatrix}, \tag{13}$$

$$\begin{bmatrix} z_{1t+1} \\ p_{2t+1} \end{bmatrix} = T(A - \tilde{B}F)T^{-1} \begin{bmatrix} z_{1t} \\ p_{2t} \end{bmatrix} + C\varepsilon_{1t+1} \tag{14}$$

where p_{2t} is an $n_2 \times 1$ vector of shadow prices associated with the non predetermined variables, z_{2t}. The matrix T provides a mapping between the state variables, z_{1t} and p_{2t}, and z_t and is given by

$$T = \begin{bmatrix} I & 0 \\ V_{21} & V_{22} \end{bmatrix}, \tag{15}$$

where V_{21} and V_{22} are submatrices of V. Finally, V and F are obtained by solving for the fix – point of

$$V = R - 2\tilde{U}F + F'\tilde{Q}F + \beta(A - \tilde{B}F)'V(A - \tilde{B}F), \tag{16}$$

$$F = (\tilde{Q} + \beta\tilde{B}'V\tilde{B})^{-1}(\tilde{U}' + \beta\tilde{B}'VA). \tag{17}$$

When the worst case misspecification is realized, the economy behaves according to equations (16) – (18). While the worst case equilibrium is certainly interesting, it is also important to consider how the economy behaves when the reference model transpires to be specified correctly. Partitioning F into $[F_u' \quad F_v']'$ where F_u and F_v are conformable with u_t and v_{t+1}, respectively. The approximating equilibrium has the form

$$z_{1t+1} = (A_{11} + A_{12}H_{21} + B_1F_{z1}^u)z_{1t} + (A_{12}H_{22} + B_1F_{p2}^u)p_{2t} + C_1\varepsilon_{1t+1}, \tag{18}$$

$$p_{2t+1} = M_{21}z_{1t} + M_{22}p_{2t}, \tag{19}$$

$$z_{2t} = H_{21}z_{1t} + H_{22}p_{2t}, \tag{20}$$

$$u_t = F_{z1}^u z_{1t} + F_{p2}^u p_{2t}, \tag{21}$$

where $H_{21} \equiv V_{22}^{-1}V_{21}$, $H_{22} \equiv V_{22}^{-1}$, $[F_{z1}^u \quad F_{p2}^u] \equiv -F_uT^{-1}$, and

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \equiv T(A - \tilde{B}F)T^{-1}. \tag{22}$$

Interestingly, the worst-case equilibrium and the approximating equilibrium share certain features. For instance, the worst-case equilibrium and the approximating equilibrium differ only with respect to the law of motion for the predetermined variables and, as a consequence, following innovations to the system the initial-period responses of the predetermined variables are the same for the approximating equilibrium as for the worst-case equilibrium. But since the decision rules for z_{2t} and u_t are also the same for the two equilibria, it follows that the initial-period responses by the nonpredetermined variables and by the policy variables are also the same. With respect to impulse response functions, differences between the approximating equilibrium and the worst-case equilibrium then only occur one period after innovations occur.

Furthermore, because the coefficient matrix on the innovations is C_1, which scales the standard deviations of the innovations, it follows that adding noise to the innovations or changing their correlation structure is not part of the evil agent's strategy. Instead, the optimally designed misspecification has the effect of changing the law of motion for the predetermined variables. More precisely, since the specification errors enter only the stochastic component of z_{1t}, the evil agent's strategy is to change the conditional means of the shock processes but not their conditional volatility.

4. ROBUST POLICY IN AN EMPIRICAL MODEL

To illustrate the robust control approach, we study the model estimated by Rudebusch ([8]), which is based on a standard New Keynesian model and contains two equations that, conditional upon the short – term interest rate, i_t, summarize the dynamics of inflation, π_t, and the dynamics of the output gap, y_t:

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1-\mu_\pi)\pi_{t-1} + \alpha y_t + \varepsilon_{\pi,t}, \qquad (23)$$

$$y_t = \mu_y E_t \pi_{t+1} + (1-\mu_y) y_{t-1} - \beta(i_t - E_t \pi_{t+1}) + \varepsilon_{y,t} \qquad (24)$$

Equation (23) is a "New Keynesian Phillips curve" derived from the optimal pricesetting behavior of firms acting under monopolistic competition, but facing price rigidities. The presence of lagged inflation and the "supply shock" $\varepsilon_{\pi,t}$ can be motivated by indexing those prices that are not reoptimized in a given period and by a time-varying elasticity of substitution across goods, leading to time-varying markups. Equation (24) can be derived from the household consumption Euler equation, where habits in consumption imply that current decisions depend to some extent on past decisions. The "demand shock" $\varepsilon_{y,t}$ can be attributed to government spending shocks or to movements in the natural level of output.8 An empirical version of this model, suitable for quarterly data and similar to that estimated by Rudebusch ([9]), is given by

$$\pi_t = \mu_\pi E_{t-1}\bar{\pi}_{t+3} + (1-\mu_\pi)\sum_{j=1}^{4}\alpha_{\pi j}\pi_{t-j} + \alpha_y y_{t-1} + \varepsilon_{\pi,t}, \qquad (25)$$

$$y_t = \mu_y E_{t-1} y_{t+1} + (1-\mu_y)\sum_{j=1}^{2}\beta_{yj} y_{t-j} - \beta_r(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + \varepsilon_{y,t} \qquad (26)$$

where $\bar{\pi}_t = 1/4\sum_{j=0}^{3}\pi_{t-j}$ is four – quarter inflation and i_t is the nominal federal funds rate (the policy instrument). We generalize the model slightly to include forward – looking behavior in the output gap equation, as in Rudebusch ([9]). The model's parameters estimates, shown in Table 1, are taken from Rudebusch ([8]) and are obtained using OLS (and survey expectations) on quarterly U.S. data from 1968:Q3 to 1996:Q4, except for the parameter μ_y, which is set to the average estimate.

Table 1 – Parameter Values

Inflation		Output		Monetary Policy	
μ_π	0.29	μ_y	0.20	β	0.99
$\alpha_{\pi 1}$	0.07	β_{y1}	1.15	λ	0.50
$\alpha_{\pi 2}$	-0.14	β_{y2}	-0.27	υ	0.10
$\alpha_{\pi 3}$	0.40	β_r	0.09		
$\alpha_{\pi 4}$	0.07	σ_y	0.833		
α_y	0.13				
σ_π	1.012				

The model's key features are that inflation and the output gap are highly persistent, that monetary policy affects the economy only with a lag, and that expectations are formed using period $t-1$ information. Notice, also, that the weights on expected future inflation and output. While consistent with much of the empirical literature, are small relative to many theory – based specifications.

The central bank's objective function is assumed to be

$$\min_{\{i_t\}} E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda y_t^2 + v i_t^2), \tag{27}$$

where we $\beta = 0.99, \lambda = 0.5, v = 0.1$. Thus, the central bank sets monetary policy to avoid volatility in inflation around its target (normalized to zero) and in the output gap around zero (precluding any discretionary inflation bias). In addition, the central bank desires to limit volatility in the nominal interest rate around target (normalized to zero). The concern for misspecification, ϕ, is chosen so that the detection error probability is 0.1, given a sample of 200 observations. This implies that $\theta = 54.5$.

The model can be written in state – space form as follows:

$$z_{t+1} = Az_t + Bu_t + C\varepsilon_{t+1}, \tag{28}$$

$$\min_{u_t} E_0 \sum_{t=0}^{\infty} \beta^t [z_t' R z_t + u_t' Q u_t], \tag{29}$$

where $z_{1t} = (\pi_t \quad \pi_{t-1} \quad \pi_{t-2} \quad \pi_{t-3} \quad y_t \quad y_{t-1})'$,

$$z_{2t} = (E_t\pi_{t+1} \quad E_t\pi_{t+2} \quad E_t\pi_{t+3} \quad E_t y_{t+1})',$$

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix},$$

$$\varepsilon_t = (\varepsilon_{\pi t} \quad \varepsilon_{yt})',$$

$$u_t = i_t,$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -6.56 & 1.37 & -3.92 & -0.69 & -1.79 & 0 & 12.79 & -1 & -1 & 0 \\ 0.74 & -0.15 & 0.44 & 0.077 & -4.4 & 1.08 & -1.44 & 0 & 0 & 0 \end{pmatrix}$$

$$B = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.45)'$$

$$C = \begin{pmatrix} 1.012 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.833 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$Q = 0.1$$

We first solved the linear quadratic optimization problem in the nonrobust case. The matrix which gives the optimal feedback is

$$K = \begin{pmatrix} 20.1 & -10.36 & 3.54 & 0.913 & -6.947 & 2.399 & -41.088 & 40.245 & -16.047 & -4.296 \end{pmatrix} \quad (33)$$

and the optimal control is:

$$u_t = i_t = F z_t = -K z_t. \quad (30)$$

Next, we solved the worst – case robust control problem. In this case,

$$u_t = \begin{pmatrix} i_t \\ v_{\pi,t+1} \\ v_{y,t+1} \end{pmatrix},$$

$$\overline{B} = \begin{pmatrix} 0 & 1.012 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.833 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.45 & 0 & 0 \end{pmatrix},$$

$$\overline{Q} = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 54.5 & 0 \\ 0 & 0 & 54.5 \end{pmatrix}.$$

Matrices A, C and R are the same as in the nonrobust case.

Solving the linear quadratic optimisation problem, we obtained the optimal feedback matrix

$$\overline{K} = \begin{pmatrix} 1.6733 & 0.992 & -1.65 & -0.302 & -9.741 & 2.3038 & -0.99 & 1.22 & -3.38 & -0.22 \\ -1.4045 & 0.783 & -0.61 & -0.125 & -0.315 & -0.0005 & 3.26 & -1.97 & 0.055 & 0.216 \\ -1.4969 & 0.472 & -0.83 & -0.152 & -0.422 & -0.0008 & 2.99 & -0.62 & -0.33 & 0.206 \end{pmatrix}$$

(31)

The optimal control is given by $\overline{u}_t = -\overline{K}z_t$, which means that the optimal policy rule and misspecification are given by:

Coefficient on

	π_t	π_{t-1}	π_{t-2}	π_{t-3}	y_t	y_{t-1}
Policy rule i_t	-1.67	-0.99	1.65	0.30	9.74	0.99
Misspeficiation						
$v_{\pi,t+1}$	1.4	-0.78	0.61	0.12	0.31	0.0005
$v_{y,t+1}$	1.49	-0.47	0.83	0.15	0.42	0.0008

In figures 1, 2, we plot impulse responses to unit – sized innovations to inflation ($\varepsilon_{\pi,t}$) under commitment using the state – space method, for the nonrobust and robust cases, respectively.

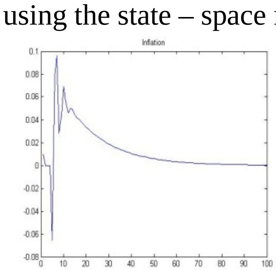

Figure 1

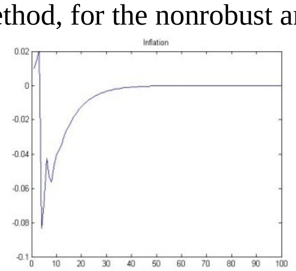

Figure 2

6. CONCLUSIONS

In formulating monetary policy, central banks must cope with substantial economic uncertainty.

Economic uncertainty can arise from different sources: the state of the economy, the nature of economic relationships, and the magnitude and persistence of ongoing shocks.
Robust control theory instructs decision makers to investigate the fragility of decision rules by conducting worst-case analyses.
In this paper we show how state space methods and structural-form solution methods can be applied to robust control problems, thereby making it easier to analyze complex models.
We illustrate the state space solution methods by applying them to an empirical New Keynesian business cycle model of the genre widely used to study monetary policy under rational expectations. A key finding from this exercise is that the strategically designed specification errors will tend to distort the Phillips curve in an effort to make inflation more persistent, and hence harder and more costly to stabilize. The optimal response to these distortions is for the central bank to become more activist in its response to shocks.

REFERENCES

[1] Backus, David K. and John Driffill (1986), "The consistency of optimal policy in stochastic rational expectations models", Discussion Paper No. 124, Centre for Economic Policy Research.

[2] Dennis, R., Leitemo, K. and Soderstrom, U., "**Methods for robust control**", Working Paper 2006 – 10, Federal Reserve Bank of San Francisco

[3] Giordani, Paolo and Paul Soderlind (2004), "**Solution of macromodels with Hansen-Sargent robust policies: Some extensions**", Journal of Economic Dynamics and Control 28 (12), 2367–2397.

[4] Hansen, Lars Peter and Thomas J. Sargent (2001), "**Robust control and model uncertainty**", American Economic Review Papers and Proceedings 91 (2), 60–66.

[5] Hansen, Lars Peter and Thomas J. Sargent (2003), "**Robust control of forward-looking models**", Journal of Monetary Economics 50 (3), 581–604.

[6] Hansen, Lars Peter and Thomas J. Sargent (2006), **Robustness**, University of Chicago and New York University, Princeton University Press.

[7] Leitemo, Kai and Ulf Soderstrom (2004), "**Robust monetary policy in the New-Keynesian framework**" Discussion Paper No. 4805, Centre for Economic Policy Research, Macroeconomic Dynamics.

[8] Rudebusch, Glenn D., "**Assessing nominal income rules for monetary policy with model and data uncertainty**", Economic Journal, 112 (479), 1–31.

[9] Rudebusch, Glenn D., "**Term structure evidence on interest rate smoothing and monetary policy inertia**" Journal of Monetary Economics, 49 (6), 1161–1187.